

Supergravity solitons and non-perturbative superstrings

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A review is given of the implications of supersymmetric black holes for the non-perturbative formulation of toroidally compactified superstrings, with particular emphasis on symmetry enhancement at special vacua and S-duality of the heterotic string.

Heterotic and type II superstrings have, when toroidally-compactified to D=4, N=4 and N=8 supersymmetry, respectively. The (generic) effective N=4 or N=8 supergravity theory has, in both cases, 28 U(1) gauge fields and hence particles in the spectrum may carry any of 56 electric or magnetic charges; we may therefore associate to each particle in the spectrum a charge 56-vector \mathcal{Z} . In perturbation theory only the 28 electric charges are carried by particles in the heterotic string spectrum, and only 12 of these occur in the perturbative type II superstring spectrum. The remaining charges are carried by particles with masses that go to infinity as the string coupling constant g goes to zero so they are absent in the free string spectrum, but they can be found by a semi-classical analysis based on the effective N=4 or N=8 supergravity theory.

Recent advances in our understanding of the non-perturbative dynamics of these theories have made essential use of the existence of this non-perturbative spectrum, with the main analytical tool being the Bogomol'nyi bound [1–3] on the mass of any particle in terms of its charge vector. All particles in the free string spectrum are, of course, found to satisfy the bound, but few of them saturate it. However, when $g \neq 0$, particles that do *not* saturate the bound can be expected to become unstable against decay into particles which do saturate it. For example, massive particles in the perturbative string spectrum which fail to saturate the bound by a large margin can be viewed as (non-extreme) black holes which will decay by Hawking radiation. It therefore seems reasonable to suppose that, at non-

zero string coupling, the only stable states are those which saturate the Bogomol'nyi bound. If this is the case, then every stable particle in the spectrum has a mass given by the formula

$$M^2 = \mathcal{Z}^T \mathcal{R}(\langle\phi\rangle) \mathcal{Z} . \quad (1)$$

where \mathcal{R} is a 56×56 matrix that depends on the expectation values $\langle\phi\rangle$ of the massless scalar fields ϕ of the effective supergravity theory, i.e. on the choice of vacuum.

In the N=8 case, the space of vacua parametrized by the scalar field expectation values $\langle\phi\rangle$ is locally isomorphic to the coset space $E_{7,7}/SU(8)$ [4], and the bound (1) is invariant under the natural action of $E_{7,7}$ on this space if \mathcal{Z} is taken to transform as the **56** of $E_{7,7}$. U-duality of the type II superstring theory [5] implies that the actual space of vacua is

$$E_7(\mathbb{Z}) \backslash E_{7,7}/SU(8) \quad (2)$$

where $E_7(\mathbb{Z})$ is the discrete U-duality subgroup of $E_{7,7}$ that contains the $S \times T$ duality group $Sl(2; \mathbb{Z}) \times SO(6, 6; \mathbb{Z})$ of the type II superstring. The matrix \mathcal{R} appearing in the N=8 Bogomol'nyi bound is non-singular, so all charged particles have non-zero masses. This is a consequence of the fact that *all* 56 charges occur as central charges in the N=8 supersymmetry algebra and is clearly necessary for the complete symmetry between all 56 types of charge implied by U-duality.

In the heterotic case, there are only 12 central charges in the N=4 supersymmetry algebra, so there must be a 44-dimensional subspace of the full 56-dimensional charge vector space for which the bound implies merely that the mass M is

non-negative. Indeed, in the heterotic case the matrix \mathcal{R} has 44 zero eigenvalues for all scalar field expectation values [6]. To investigate the nature of the zero-eigenvalue eigenspace it is convenient to split the scalar field expectation values, which we have denoted generically by $\langle\phi\rangle$, into the complex axion/dilaton expectation value $\lambda = \theta/2\pi + i/g^2$, where g is the string coupling constant and θ a vacuum angle, and the remaining moduli φ , which take values in the coset space $SO(6, 22)/[SO(6) \times SO(22)]$. We can then set $\mathcal{Z}^T = (p, q)$, where p denotes the magnetic charges and q the electric ones, and rewrite the mass formula (1) in the form [7]

$$M^2 = (p \quad q) [\mathcal{S}(\lambda) \otimes \mathcal{T}(\varphi)] \begin{pmatrix} p \\ q \end{pmatrix}, \quad (3)$$

where the matrix \mathcal{T} acts on the electric and magnetic 28-vectors q and p , and the 2×2 matrix \mathcal{S} acts on the 2-vector (p, q) . This mass formula is invariant under the action of

$$Sl(2; \mathbb{R}) \times SO(6, 22) \quad (4)$$

on the charge 56-vector (p, q) provided that the moduli λ and φ are simultaneously transformed. We note that T-duality of the heterotic string theory implies that the space parameterized by the moduli φ is actually

$$SO(6, 22; \mathbb{Z}) \backslash SO(6, 22) / [SO(6) \times SO(22)], \quad (5)$$

i.e. vacua which differ by an $SO(6, 22; \mathbb{Z})$ transformation are equivalent. Similarly, S-duality [8] implies that the space parameterised by λ is actually

$$Sl(2; \mathbb{Z}) \backslash Sl(2; \mathbb{R}) / U(1). \quad (6)$$

Unlike S-duality, which is invisible in perturbation theory, T-duality has consequences for the perturbative heterotic or type II superstring. These predictions of T-duality have been verified (see [9] for a review) but this is insufficient to establish T-duality as a property of the full non-perturbative superstring theory; at this level S, T and U duality are *all* conjectural, although evidence in their support continues to mount.

The matrix \mathcal{S} in (3) is non-singular for all values of λ . The reason that the 56×56 matrix \mathcal{R}

has 44 zero eigenvalues is that the 28×28 matrix \mathcal{T} has 22 zero eigenvalues. Let us concentrate on the electric charge 28-vectors q . They can be classified according to whether their $SO(6, 22)$ norm q^2 is positive, zero or negative. The 22-dimensional kernel of \mathcal{T} is spanned by *negative* norm 28-vectors, but a negative norm is only a necessary, and not a sufficient, condition for a vector to lie in the kernel of \mathcal{T} . Of course, given a negative-norm charge vector that is not in this kernel, there is an $SO(6, 22)$ transformation that will take it into the kernel. However, this transformation will also change the vacuum. Thus, for a given vacuum, a purely electric particle with a *given* negative norm electric charge vector will not generally be massless, but there will be vacua in which it *is* massless [6]. Instead of changing the vacuum we could just start with a different negative norm charge vector (the possible choices differ by $SO(6, 22)$ transformations) and it might appear from this that charged massless particles must occur for *any* choice of vacuum. But we have yet to take into account the charge quantization required by quantum mechanics.

Charge quantization requires, for any given vacuum, that the charge vector lie in a lattice. An $SO(6, 22)$ transformation of a charge vector (for fixed vacuum) will generally take a vector in this lattice to one that is not in the lattice. In fact, it will do so unless the $SO(6, 22)$ transformation is actually an $SO(6, 22; \mathbb{Z})$ transformation, where the embedding of the discrete subgroup $SO(6, 22; \mathbb{Z})$ in $SO(6, 22; \mathbb{R})$ depends on the vacuum. Thus, the allowed choices of initial negative norm charge vectors belong to a discrete, rather than a continuous, set. This set is the union of $SO(6, 22; \mathbb{Z})$ orbits of lattice vectors, each orbit being characterised by an $SO(6, 22)$ norm.

Generically, none of these allowed charge vectors will be in the kernel of \mathcal{T} but, as explained above, there will be vacua in which any one of them is. If there were particles in the spectrum with arbitrarily large negative q^2 , then these special vacua would be dense in the space of all vacua. It is difficult to see how one could make physical sense of this. Fortunately, this doesn't happen because the 'Bogomol'nyi' states in the perturbative heterotic string spectrum all have

$q^2 = -2 + 2N_L$ where the integer N_L is the oscillator number of the left moving worldsheet fields (see, for example, [10]). Since N_L is non-negative, the only perturbative states with negative q^2 are those for which $N_L = 0$, which have $q^2 = -2$. In this context, the fact that there exist special vacua in which these states are massless is well-known as the Halpern-Frenkel-Kač mechanism.

It seems possible that this restriction on the allowed negative values of q^2 may have a general explanation (as just remarked, the absence of any restriction leads to paradoxical conclusions) but this explanation is lacking at present. Since we have invoked results already established in perturbation theory to restrict the possible charge vectors, it might seem that little has been gained from the prior general analysis based on the Bogomol'nyi mass formula. However, the mass formula does give additional information because it tells us that a vacuum in which electrically charged particles are massless is also a vacuum in which magnetically charged particles are massless [6]. In fact, if S-duality is assumed, each additional massless particle will be one member of an entire $SL(2; \mathbb{Z})$ orbit of additional massless particles.

This explains an otherwise puzzling feature of the HFK mechanism. T-duality requires that the massless effective field theory have an $SO(6, 22)$ invariance of the equations of motion. The perturbative HFK mechanism suggests that the effective field theory in a vacuum with enhanced symmetry is a locally supersymmetric, rank 28, N=4 non-abelian gauge theory, but such theories are *not* $SO(6, 22)$ invariant. The resolution is that the evidence of perturbation theory is misleading because the fact that magnetically charged particles must become massless simultaneously with the electric ones means that the physics in these enhanced symmetry vacua cannot be described by a local field theory.

Another obvious advantage of the approach to symmetry enhancement via the Bogomol'nyi mass formula is that it depends only on N=4 supersymmetry and therefore applies equally to the $K_3 \times T^2$ compactified type II superstring [6], thus providing evidence for the proposal [5] that these two string theories are non-perturbatively equivalent.

That the particles in the spectrum with negative q^2 are the ones responsible for enhanced symmetry at special vacua is not surprising when one considers that by switching off the supergravitational interactions we remove the six gauge fields in the graviton multiplet and break the $SO(6, 22)$ invariance to $SO(22)$. We then have an N=4 $U(1)^{22}$ gauge theory coupled to massive charged N=4 supermultiplets for which the charge vectors automatically have negative norm. These multiplets have a natural interpretation as Higgs supermultiplets. Their magnetic duals are BPS monopoles [10]. In this 'Higgs' sector the conjectured S-duality of the heterotic string can therefore be viewed as generalization of Montonen-Olive duality [11,12].

The connection between field theory solitons and particles in the spectrum of the quantum theory is usually made via semi-classical quantization. These methods are generally reliable at weak coupling, but one could question their applicability near those special vacua of the heterotic string at which otherwise massive solitons become massless. Indeed, one might well expect the whole soliton picture to break down at these points. It is therefore rather surprising to learn that this does not happen, in the sense that *massless* 'soliton' solutions can be found [13,14]. They are singular, however, and their significance for the quantum theory has still to be elucidated.

We now move on to the electric states with $q^2 \geq 0$. Since only the $q^2 = -2$ states involve the (broken) non-abelian group structure one might suspect that these should appear as solutions of the effective massless $U(1)^{28}$ supergravity theory. To explore this possibility it is convenient to consider the consistent truncation of the effective supergravity theory of the heterotic string to one with the Lagrangian

$$L = \sqrt{-g} \left[R - 2(\partial\sigma)^2 - e^{-2a\sigma} F^2 \right] \quad (7)$$

where $F = dA$ is a Maxwell field-strength two-form and σ is a scalar field which is some function of the scalars ϕ ; i.e. some combination of the dilaton and the other moduli fields. We may assume without loss of generality that a is positive since the field redefinition $\sigma \rightarrow -\sigma$ effectively changes

the sign of a . The only values of a that arise in this way are [5]

$$a = 0, \frac{1}{\sqrt{3}}, 1, \sqrt{3}. \quad (8)$$

Strictly speaking, the value $a = 0$ does not arise in this way, but rather via a consistent truncation to N=2 supergravity in which all scalars are absent, but bosonic solutions of N=2 supergravity may be considered as solutions of the field equations of (7) with constant σ when $a = 0$. The values $a = 1/\sqrt{3}$ and $a = \sqrt{3}$ arise from a truncation to D=5 supergravity followed by dimensional reduction to D=4. Since there is already a vector field in the D=5 graviton supermultiplet, the resulting D=4 Lagrangian has *two* vector fields, and a further consistent truncation to one of them yields the above two values of a . As is clear from the $D = 5$ origin of the Lagrangian (7) when $a = \sqrt{3}$ or $a = 1/\sqrt{3}$, the scalar field σ is *not* the dilaton in this case but rather a modulus field whose expectation value is related to the radius of the extra dimension. The value $a = 1$ arises from a consistent truncation to N=4 supergravity, in which case σ *is* the dilaton.

The above discussion raises the following puzzle, which we now pause to resolve. Clearly, the Lagrangian of (7) with $a = 1$ cannot be *consistently* truncated to Maxwell-Einstein theory, but Maxwell-Einstein theory is the bosonic sector of N=2 supergravity which *is* a consistent truncation of N=4 supergravity. There is no immediate contradiction here because (7) with $a = 1$ is not the bosonic Lagrangian of N=4 supergravity, but it fails to be so only by virtue of the fact that we have discarded five of the six vector fields. Since the same Lagrangian is found from discarding *any* five of the six, it would seem that the inclusion of the other five cannot make a difference. However, we should here recall that the vector fields of N=4 supergravity can couple to the dilaton with either $a = 1$ or $a = -1$, since a duality transformation changes the sign of a . As long as we have only *one* vector field the sign is irrelevant but with more than one it is not. In fact, the truncation to N=2 supergravity involves the identification of two vector fields with opposite sign dilaton coupling constants.

Before continuing it is worthwhile to consider the limitations of the truncation to (7). Although the consistency of the truncation ensures that any solution of the truncated field equations is a solution of the full field equations (this is what is meant by the adjective ‘consistent’ in this context), the converse is not true; i.e. there will certainly be solutions of the full untruncated equations that cannot be found among the solutions of the truncated equations. These include dyonic black holes for $a \neq 0$, which require a non-vanishing axion field in addition to the scalar field σ . Thus, within the context of (7) we can at best verify predictions of a \mathbb{Z}_2 subgroup of the S-duality group $Sl(2; \mathbb{Z})$. However, these solutions can be obtained [15,16] (at least for $a=1$) by an S-duality transformation of the solutions with vanishing axion field. Another class of black hole solutions that cannot be found among solutions of the truncated equations have been found in [17]; these have also been called ‘dyonic’ but it should be noted that they do not carry both electric and magnetic charge *of the same type*. In the context of N=8 supergravity these ‘dyonic’ solutions always break more than half the supersymmetry and are therefore not part of the Bogomol’nyi spectrum in the sense used here. To the extent that such solutions break half the N=4 supersymmetry of the heterotic string the discussion given here in terms of the truncated Lagrangian (7) must be considered incomplete.

To make the connection between solutions of the field equations of the truncated Lagrangian (7) and electrically charged particles in the heterotic string spectrum we observe that for the truncated Lagrangian (7) the Bogomolnyi bound reduces to

$$M^2 \geq \frac{1}{(1+a^2)} [e^{2a\langle\sigma\rangle} Q^2 + e^{-2a\langle\sigma\rangle} P^2] \quad (9)$$

where Q and P are, respectively, the electric and the magnetic charge associated with the gauge field A of (7). Clearly, Q is some combination of the 28 electric charges and P some combination of the 28 magnetic charges, the combination depending on the precise nature of the truncation leading to (7). Extreme black hole solutions saturating the bound (9) exist for all values of a

[18–20], but they have special properties for the particular values of (8), some of which will be mentioned below.

Actually, ‘black hole’ is an abuse of terminology since many of the solutions we shall be concerned with are neither ‘black’ nor ‘holes’. Generally we would expect particles in the perturbative string spectrum to appear as naked singularities (for otherwise they would have a classical structure incompatible with their interpretation as excitations of a fundamental, i.e. structureless, string) and we would similarly expect solutions representing particles in the non-perturbative spectrum to appear as non-singular solitons (otherwise, there is no obvious justification for their inclusion in the spectrum). There is no single term that covers both these cases, not to mention several others, so ‘black hole’ will have to do.

If the mass formula for electric black holes, obtained by saturation of the bound (9), is now compared to the mass formula of the perturbative heterotic string spectrum for $\langle\sigma\rangle = 0$ then the result is that the $a = \sqrt{3}$ extreme electric ‘black holes’ can be identified with the $q^2 = 0$ states and $a = 1$ extreme electric black holes with the $q^2 \geq 0$ states [21]. As anticipated, these all have naked timelike singularities. The possible roles of the $a = 0$ and $a = 1/\sqrt{3}$ extreme black holes will be discussed later.

The $q^2 = 0$ states are the supermultiplets of Kaluza-Klein (KK) and winding mode states with maximum spin 2. Since these modes involve an extra dimension in an essential way one might expect the electric $a = \sqrt{3}$ black holes to have a D=5 interpretation. In fact, there should be two such interpretations, one for the KK modes and the other for the winding modes. The KK interpretation arises from the fact that the electric $a = \sqrt{3}$ black holes can be interpreted as pp waves moving in the fifth dimension [22]; the corresponding quanta can therefore be interpreted as particles moving at the speed of light in the fifth dimension, but this is essentially a description of KK states. The other interpretation is as a D=5 string. There are actually *three* D=5 string solutions of the effective action of the D=5 heterotic string, which double-dimensionally reduce to extreme black holes with $a = 1/\sqrt{3}$, $a = 1$ and

$a = \sqrt{3}$ [23]. Obviously, the last case is the one of relevance here.

Since the long range fields of the $q^2 = 0$ states are those of extreme electric $a = \sqrt{3}$ black holes their magnetic duals must be the magnetic $a = \sqrt{3}$ black holes. These are singular in D=4 but can be interpreted either as KK monopoles which are non-singular in D=5 [24,25] or as (abelian) H-monopoles [26,27]. The non-singularity of these solutions is consistent with their interpretation as solitons. Note that the electromagnetic \mathbb{Z}_2 subgroup of the S-duality group does *not* exchange the KK states with the KK monopoles and the string winding states with the H-monopoles, as one might expect but rather the other way around. That is, the KK states are exchanged with the H-monopoles and the winding modes with the KK monopoles. The reason for this is that the KK states and the H-monopoles both have $M \sim 1/R$, where R is the radius of the fifth dimension while the winding modes and the KK monopoles have $M \sim R$, and S-duality does not affect R . Thus, ‘pyrgon/KK-monopole duality’ [22,28] in the KK sector is a consequence of the *combined* S and T-dualities of the heterotic string theory.

In the type II case, there is a simple consideration [5] which shows that *all* charged particles with masses given by the Bogomol’nyi mass formula, i.e. those breaking half the N=8 supersymmetry, must correspond to $a = \sqrt{3}$ extreme black holes. The moduli space of multi-black hole solutions breaking half the N=8 supersymmetry must be flat, but this is the case if and only if $a = \sqrt{3}$ [29,30]. As in the heterotic case, some of these black holes can be interpreted as fundamental string states, of either KK or string winding type (and this identification is required by U-duality). The remainder can all be interpreted as (non-perturbative) p-brane ‘wrapping modes’ [5]. This interpretation becomes particularly simple in D=11. I will not go into this here as I have recently reviewed the role of p-branes and D=11 supergravity elsewhere [31].

It remains to consider the Bogomolnyi mass spectrum of the heterotic string with $q^2 > 0$. Recall that the long range fields of these states must be those of $a = 1$ extreme black holes. As confir-

mation of this identification, we note that, since a charge vector with positive $SO(6, 22)$ norm can be rotated to one that is carried only by vector fields in the graviton supermultiplet, this sector of the Bogomol'nyi spectrum is essentially of purely gravitational origin. This leads one to expect these particles to appear as extreme black hole solutions of N=4 supergravity, for which the 'dilaton' coupling constant is indeed $a = 1$. Moduli space considerations provide a check on this because the moduli space of solutions of an $N = 4$ supersymmetric field theory which break half the supersymmetry must be hyper-Kähler [32]. At first sight, this condition appears not to be fulfilled because the moduli space of $a = 1$ black hole solutions of the field equations of (7) has the wrong dimension to be hyper-Kähler, but it must be remembered that the Lagrangian (7) is *not* the bosonic sector of an N=4 supergravity theory, even for $a = 1$, but rather a consistent truncation of one. It follows that the moduli space need not be hyper-Kähler but must be a totally geodesic submanifold of a hyper-Kähler manifold, as indeed it is [33].

One might wonder why an $a = 1$ black hole cannot decay into two $a = \sqrt{3}$ black holes given that, according to the mass formula (9), the mass of an $a = 1$ black hole with charge Q is greater than the sum of any number of $a = \sqrt{3}$ extreme black holes with total charge Q . This is the wrong comparison, however, since the charge carried by an $a = 1$ black hole cannot be identified with the charge carried by the $a = \sqrt{3}$ black holes. The point is that $a = \sqrt{3}$ extreme electric black holes have $q^2 = 0$, so that they must carry more than one charge of the untruncated theory and, consequently, the addition of charges is *vector* addition. Consider the decay of an $a = 1$ extreme black hole of charge $Q_{(1)}$ into two $a = \sqrt{3}$ extreme black holes, each of charge $Q_{(\sqrt{3})}$; in order that the charge vectors of the latter sum to the charge vector of the former they must be orthogonal, so charge conservation implies that $Q_{(1)} = \sqrt{2}Q_{(\sqrt{3})}$, rather than $Q_{(1)} = 2Q_{(\sqrt{3})}$. Application of the mass formula (9) now shows that although the decay is still energetically possible it has no available phase space and so cannot pro-

ceed.

As noted earlier, in the $a = 1$ case the scalar σ in (7) is the dilaton so $g = e^{\langle\sigma\rangle}$ is the string coupling constant and we can therefore rewrite the mass formula (9) as

$$M^2 = \frac{1}{2}[g^2 Q^2 + g^{-2} P^2] . \quad (10)$$

But this cannot be right in the string theory context because electrically charged particles would all have zero mass at $g = 0$. The resolution [34,35] involves the realization that the Lagrangian from which (10) was derived was expressed in terms of the canonical, or 'Einstein', metric ds^2 , whereas we should be using the *string metric* $d\tilde{s}^2 = g^2 ds^2$. A simple way to obtain the formula appropriate to the string metric is to observe that the ADM mass M of an asymptotically-flat spacetime depends implicitly on the normalization at spatial infinity, $\lim(-k^2)$, of an asymptotic timelike Killing vector field k . To make this dependence explicit we must replace (10) by

$$M^2 = \frac{1}{2}[g^2 Q^2 + g^{-2} P^2] \lim(-k^2) . \quad (11)$$

Normally we choose $\lim(-k^2) = 1$ where k^2 is defined in terms of the 'Einstein' metric, but in the string theory context we should instead define $\lim(-\tilde{k}^2) = 1$ where \tilde{k}^2 is defined in terms of the string metric. Since $\tilde{k}^2 = g^2 k^2$ we should set $\lim(-k^2) = g^{-2}$ in (11) to get

$$M^2 = Q^2 + \frac{1}{g^4} P^2 , \quad (12)$$

where the electrically charged particles now have masses that are independent of g , as is appropriate for particles in the perturbative string spectrum, while the magnetically charged particles have masses with the g^{-2} dependence expected of solitons.

In the type II case the formula analogous to (12) is, *schematically*,

$$M^2 = q^2 + \frac{1}{g^4} p^2 + \frac{1}{g^2} r^2 , \quad (13)$$

where q and p now represent, respectively, the NS-NS electric and magnetic charges and r represents a RR charge, either electric or magnetic.

As the formula shows, both magnetic *and* electric RR charges can appear only non-perturbatively as required by U-duality [5], but their masses are larger than is usual for solitons by a factor of g^{-1} [34]. As mentioned earlier, only the $a = \sqrt{3}$ black holes saturate the Bogomol'nyi bound in the type II case because only these preserve half the supersymmetry. The $a = 0$ black holes are, of course, also solutions of N=8 supergravity, because they were solutions for N=4, but in the N=8 context they preserve only 1/4 of the supersymmetry. Presumably they will therefore be unstable against decay into $a = \sqrt{3}$ black holes. The same argument can be used to dispose of the $a = 0$ and $a = 1/\sqrt{3}$ black holes in the N=4 case since both preserve only 1/4 of the N=4 supersymmetry. The values of a for which the extreme black holes saturate the N-extended Bogomol'nyi bound is shown in the table below.

Table 1
Bogomol'nyi Black Holes

No. of Susys	Scalar Coupling
N=8	$a^2 = 3$
N=4	$a^2 = 1, 3$
N=2	$a^2 = 0, \frac{1}{3}, 1, 3$

Thus, as long as we are concerned with stable particles of $N = 4$ and $N = 8$ superstring theories we need not consider the $a = 0$ and $a = 1/\sqrt{3}$ cases. Nevertheless, it is interesting to consider what their study might tell us about N=2 superstring compactifications, although one should bear in mind that renormalization effects might change the picture.

Consider first the $a = 0$ extreme black holes. The first point to note is that the electric and magnetic cases have an *identical* metric, the extreme Reissner-Nordstrom metric. So, on the evidence of supergravity solutions, both should be given the *same* interpretation, either fundamental or solitonic. It is impossible for both to be (simultaneously) fundamental so a solitonic interpretation of both the magnetic *and* the electric $a = 0$ black holes is the only option. This would

make them similar to RR solitons in type II superstring theories. However, the RN metric has a timelike singularity, admittedly hidden behind an event horizon, so the soliton interpretation is problematic and there is therefore no really compelling reason to include $a = 0$ black holes as part of a superstring spectrum. It is instructive to compare this situation with that of $a = 1$ extreme black holes. The Einstein metric is again the same for both the electric and magnetic cases but the relevant metric is now the string metric, in terms of which the electric and magnetic solutions differ radically: the electric $a = 1$ black hole has a naked timelike singularity, in accord with its fundamental status, while the magnetic $a = 1$ black hole is geodesically complete [20], in accord with its solitonic status.

Since the Lagrangian (7) has a D=5 interpretation for $a = 1/\sqrt{3}$, one should not be surprised to discover that the $a = 1/\sqrt{3}$ black holes have a D=5 interpretation. Indeed, the electric extreme $a = 1/\sqrt{3}$ black hole is just a dimensional reduction of the D=5 RN black hole [36,37] while the magnetic extreme $a = 1/\sqrt{3}$ black hole is interpretable as a double dimensional reduction of a D=5 string [38]. The global structure of these solutions is more promising than for the $a = 0$ case because although both the D=5 black hole and the D=5 string have event horizons, the string is nevertheless geodesically complete. The situation here is entirely analogous to that of the electric membrane and magnetic fivebrane of D=11 supergravity [39,38], which arise in a similar manner from the D=10 type IIA electric membrane and magnetic fourbrane. This is but one aspect of the quite close analogy between D=5 and D=11 that may well repay future study.

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